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Numerics of Partial Differential Equations – Tutorial 10

Exercise 10.1 [Convergence Richardson-Iteration]

Let A be a symmetric and positive definite matrix with real eigenvalues, which are ordered. Show that the Richardson-iteration converges if and only if

$$0 < \omega < \frac{2}{\lambda_n}$$

holds.
Additionally, show that the optimal relaxation parameter ω_{opt} is given by

$$\omega_{\text{opt}} = \frac{2}{\lambda_1 + \lambda_n}$$

with the spectral radius

$$\rho(B) = \frac{\lambda_n - \lambda_1}{\lambda_n + \lambda_1}$$

of the iteration matrix $B = I - \omega_{\text{opt}} A$.

Home Assignment 10.2 [Multigrid-Cycles]

Write the first step of algorithm 10.29 (geometric multigrid, GMG) for $j = 2$ and the cases $\gamma = 1, 2$ explicitly. This means that all recursions should be written explicitly without resorting to just calling the recursive functions.
Additionally, graph the path of the algorithm with respect to the different levels. The levels j should be on the y axis. Only show the steps restriction, prolongation, smoothing and solving using arrows
Note: The loops over ν_1 and ν_2 can be simplified as the methods `presmooth()` and `postsmooth()`

Home Assignment 10.3 [Smoothing Property Twogrid-Method Richardson]

Note: *For this assignment you need the basics of chapter 10.7.5. Especially the definitions of the norms are needed.*
Let A be s.p.d.
Show that the Richardson-iteration $x_h^{k+1} = x_h^k + \omega(b_h - A_h x_h^k)$ with $\omega = 1/\lambda_{\max}(A_h)$ fulfills

$$\|x_h - x_h^\nu\|_2 \le \frac{\lambda_{\max}(A_h)}{\nu} \|x_h - x_h^0\|_0$$