Institut für Angewandte Mathematik

Leibniz Universität Hannover

Prof. Dr. Thomas Wick

M.Sc. Philipp Thiele

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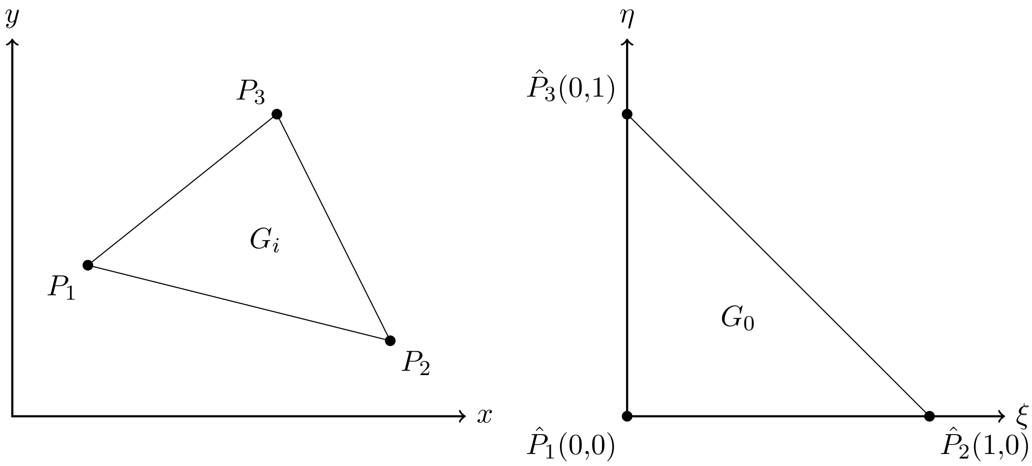
Numerics of Partial Differential Equations – Tutorial 5

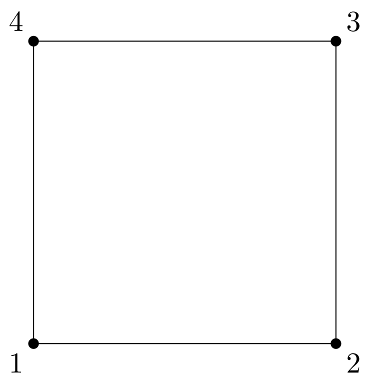
Exercise 5.1 [Master Element of a Triangle]



Derive a Transformation $T\colon (\xi, \eta) \mapsto (x, y)$ from the unit triangle G_0 to a general triangle G_i.

Exercise 5.2 [Mass-Matrix of a Bilinear Element]

Given the unit square $[0,1] \times [0,1]$ with DoFs in the corners.



The (bi-)linear basis functions can be derived by multiplying the corresponding linear basis functions for x and y respectively:

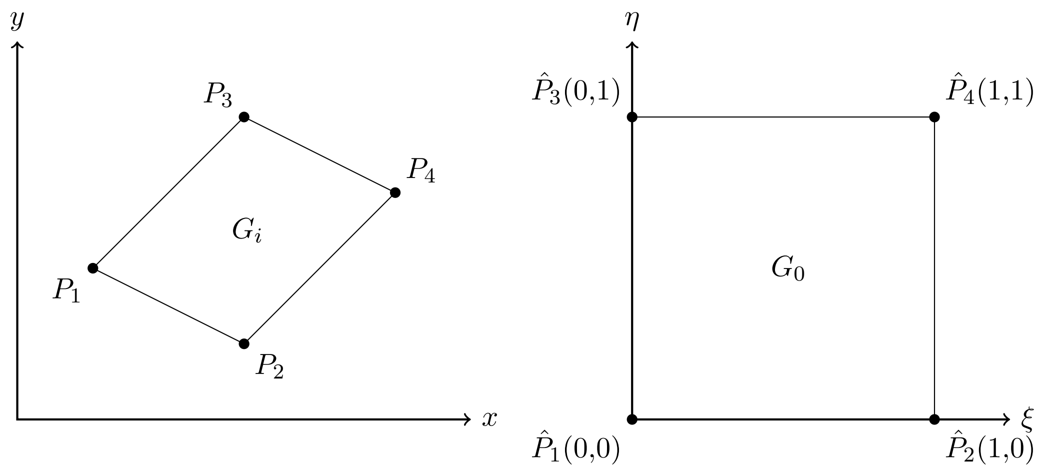
$$\psi_1^x = x, \quad \psi_2^x = (1-x)$$
$$\psi_1^y = y, \quad \psi_2^y = (1-y)$$
$$\Longrightarrow$$
$$\varphi_1(x,y) := \psi_1^x(x) \cdot \psi_1^y(y)$$
$$\varphi_2(x,y) := \psi_2^x(x) \cdot \psi_1^y(y)$$
$$\varphi_3(x,y) := \psi_2^x(x) \cdot \psi_2^y(y)$$
$$\varphi_4(x,y) := \psi_1^x(x) \cdot \psi_2^y(y)$$

Using Simpson's rule, calculate the so called mass-matrix

$$M_{i,j} = \int_0^1 \int_0^1 \varphi_i(x,y) \cdot \varphi_j(x,y) dx\, dy$$

Hint: Use symmetry and further properties of the integrals to simplify the calculation.

Home Assignment 5.3 [Master Element of a Parallelogram]



(a) Derive a Transformation $T\colon (\xi,\eta) \mapsto (x,y)$ from the unit square G_0 to a general parallelogram G_i.

(b) Calculate the Jacobian determinant J, that is needed for the integration by substitution. Namely for transforming the integral measure from G_i to G_0

$$dxdy = J\; d\xi d\eta.$$

(c) Using T and J transform the following integrals into integrals over the unit square G_0.

$$\iint_{G_i} \varphi\; dxdy \tag{1}$$

$$\iint_{G_i} \varphi^2\; dxdy \tag{2}$$

$$\iint_{G_i} \langle \nabla\varphi, \nabla\varphi \rangle\; dxdy = \iint_{G_i} \frac{\partial\varphi}{\partial x}^2 + \frac{\partial\varphi}{\partial y}^2\; dxdy \tag{3}$$

Home Assignment 5.4 [Stiffness-Matrix of a Bilinear Element]

Given the bilinear unit-element $[0,1]\times[0,1]$, as in the previous exercise.
Using Simpson's rule, calculate the so called stiffness-matrix

$$K_{i,j} = \int_0^1 \int_0^1 \langle \nabla\varphi_i(x,y), \nabla\varphi_j(x,y) \rangle \, dx\, dy$$

Hint: Use symmetry and further properties of the integrals to simplify the calculation.