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Numerics of Partial Differential Equations – Tutorial 7

Exercise 7.1 [weak formulation with inhomogeneous Neumann boundary]

Given Poisson's Problem on the following domain $\Omega = [0,1] \times [0,1]$:

$$\begin{aligned} -\Delta u = f &\quad \text{in} \quad \Omega \\ u = 0 &\quad \text{on} \quad \Gamma_D \\ \partial_n u = g &\quad \text{on} \quad \Gamma_N \end{aligned}$$

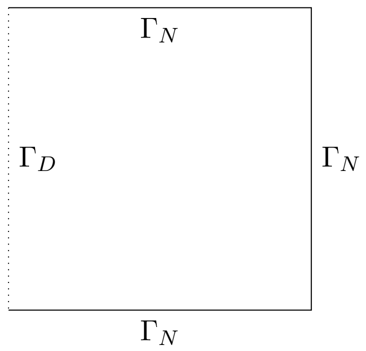

Figure 1: domain Ω

Using integration by parts derive the weak formulation of this problem.
Hint: split the boundary integral into the two boundary parts.

Exercise 7.2

On Cauchy-Sequences. Let $\Omega = (0,2)$.
We investigate the function space $C(\Omega)$ with corresponding L_2-norm

$$\|u\|_{L^2} := \left(\int_\Omega u^2 \, dx \right)^{1/2}$$

Let $(u_n)_{n\in N} \subset C(\Omega)$ be a sequence of functions satisfying

$$u_n(x) = \min\{1, x^n\}.$$

(a) Show that indeed, (u_n) is a Cauchy-sequence.

(b) Show that the limit u is not an element of $\{C(\Omega), \|\cdot\|\}$.

(c) What does this result mean from the viewpoint of functional analysis?

Home Assignment 7.3 [weak formulation with inhomogeneous Neumann boundary]

Given Poisson's Problem on the following domain $\Omega = [0,1] \times [0,1]$:

$$\begin{aligned} -\Delta u = f &\quad \text{in} \quad \Omega \\ u = 0 &\quad \text{on} \quad \Gamma_D \\ a\,u + b\,\partial_n u = h &\quad \text{on} \quad \Gamma_R \end{aligned}$$

Using integration by parts derive the weak formulation of this problem.

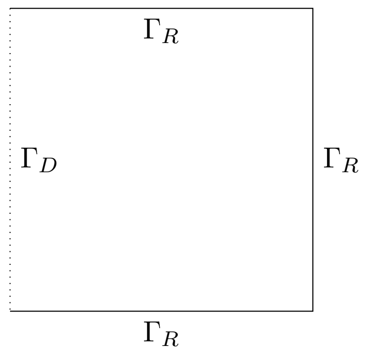

Figure 2: domain Ω

Hints:
Rearrange the boundary condition with respect to $\partial_n u$.
Split the boundary integral into the two boundary parts.
In the end all integrals containing u should be on the left hand side and all other integrals on the right hand side.

Home Assignment 7.4

Show that C_0^∞ is a dense subset of $L^p(\Omega)$. That means for every $f \in L^p(\Omega)$ there exists a sequence $(u_n)_{n\in N} \subset C_0^\infty$ satisfying $\|u_n - f\| \to 0$ for $n \to \infty$.