



Numerics of Partial Differential Equations – Tutorial 8

Exercise 8.1

Let Ω be bounded by a n -dimensional hypercube with sides of length s . With this, show that

$$\|u\|_{L^2} \leq s|u|_{H^1} \quad \forall u \in H_0^1(\Omega),$$

holds.

Exercise 8.2

Let $\Omega \subset \mathbb{R}^2$ be the unit circle D and

$$u(x, y) = \log \log \frac{2}{r}$$

with $r^2 = x^2 + y^2$. Show that $u \in H^1(\Omega)$, but u is not continuous. The results from this exercise shows that in higher dimensions we no longer have the exactness of the FEM solution at the support points.

Home Assignment 8.3

Show that the space of polynomials of degree k , i.e. P_k , is **dense** in the space of continuous functions $C([a, b])$.

Home Assignment 8.4

Let $\Omega = [a, b] \subset \mathbb{R}$, then it holds that $H^1(\Omega) \subset C(\Omega)$.